



TITLE:

Group Duality and the Kubo-Martin-Schwinger Condition (作用素環の研究とその応用)

AUTHOR(S):

MASAMICHI, TAKESAKI; KASTLER, DANIEL

CITATION:

MASAMICHI, TAKESAKI ...[et al]. Group Duality and the Kubo-Martin-Schwinger Condition (作用素環の研究とその応用). 数理解析研究所講究録 1979, 356: 22-23

ISSUE DATE:

1979-06

URL:

<http://hdl.handle.net/2433/104473>

RIGHT:

Group Duality and the Kubo-Martin-Schwinger Condition

Masamichi Takesaki

Daniel Kastler

(presented by D. K.)

For ω an α -invariant state of a C^* -algebra \mathcal{A} endowed with an action $g \in G \rightarrow \alpha_g \in \text{Aut } \mathcal{A}$ of a locally compact abelian group G , we study the relationship between the two following facts:

- (i) ω is KMS w.r.t. some continuous 1-parameter subgroup of G
- (ii) there exists a linear map T_0 , closable in an appropriate topology, such that

$$T_0 F_{AB} = G_{AB}, \quad A, B \in \mathcal{A}, \quad \text{where } F_{AB}(g) = \omega(B\alpha_g(A)),$$

$$G_{AB}(g) = \omega(\alpha_g(A)B), \quad g \in G.$$

1) Typical result: Assume ω extremal τ -invariant for an action τ of an amenable group H on \mathcal{A} which is asymptotically abelian and commutes with α ; and imbed the set $\mathcal{F} = \{F_{AB}; A, B \in \mathcal{A}\}$ in either $L^\infty(G)$ or $B(G)$ endowed with their weak*-topologies. If there is a closable linear map T_0 s.t. $T_0 F_{AB} = G_{AB}$ for all $A, B \in \mathcal{A}$, (i) holds.

Ingredients are:

2) Duality results; e.g: Let \mathcal{D} be a weak*-dense translation invariant subalgebra of $L^\infty(G)$ (or $B(G)$). If T is a weak*-closed homomorphism of \mathcal{D} into $L^\infty(G)$ (or $B(G)$) commuting with the translations, there is a continuous 1-parameter group $\{g(t), t \in \mathbb{R}\}$ of G and $s_0 \in G$ s.t. for each $f \in \mathcal{D}$ and $s \in G$, $t \rightarrow f(s + g(t))$ and $t \rightarrow Tf(s + s_0 + g(t))$ are the boundary values of a function in $H^\infty(\{| \text{Im} z | \leq 1\})$.

- 3) Density results, e.g. : Let ω be an α -invariant state of \mathcal{A} generating the covariant representation $\{\pi, H\}$. Then
- (i) \mathcal{F} is weak $*$ -total in $B(G)$ iff $\text{Sp}U = \hat{G}$
 - (ii) Assuming ω extremal τ -invariant for an action τ of an amenable group H on \mathcal{A} which is asymptotically abelian and commutes with α , \mathcal{F} is weak $*$ -total in $L^\infty(G)$.

Results of the type 3) allow to apply results of the type i) to the closure T of the operator T_0 in results like 1) to obtain the conclusion.

Bibliography

- [1] R. Haag, N.M. Hugenholtz, M. Winnink. On The Equilibrium States in Quantum Statistical Mechanics, Comm. Math. Phys. 5(1967) 215.
- [2] M. Takesaki. Tomita's Theory of Modular Hilbert Algebras and its Applications, Springer Lecture Notes in Mathematics, No. 128.
- [3] H. Araki, R. Haag, D. Kastler, M. Takesaki. Extension of States and Chemical Potential, Comm. Math. Phys. 53 (1977), 97.
- [4] N. Tatsuuma. An Extension of AKTH-theory to Locally Compact Groups, Kôkyûroku, Research Inst. Math. Sci. Kyôto Univ. 314(1977), 88-104.
- [5] M. Takesaki, N. Tatsuuma. Duality and Subgroups, Annals of Math. 93(1971), 344.